S-1448

F. E. (SEMESTER – I) EXAMINATION,

ENGINEERING MATHEMATICS – I (Revised) 59177

SUB CODE : 59177

DAY AND DATE : Friday 20/12/13 TOTAL MARKS : 100

TIME : 10.00 to 01.00 p.m.

INSTRUCTIONS : 1) All Questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of non – programmable calculator is allowed.

SECTION – I

Q. 1. Attempt ANY THREE (15)

a) Reduce the following matrix to the normal form and hence find its rank

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

b) Test for consistency and if possible solve

$$x_1 + 2x_2 - x_3 = 1$$
$$3x_1 - 2x_2 + 2x_3 = 2$$
$$7x_1 - 2x_2 + 3x_3 = 5$$

c) Investigate for what values of λ and μ the system of simultaneous equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have an infinite number of solutions.

d) Solve the following system of equations

$$x + y + 2z = 0$$
$$x + 2y + 3z = 0$$
$$x + 3y + 4z = 0$$
$$3x + 4y + 7z = 0$$

Q. 2. Attempt ANY THREE (15)

a) Examine the following set of vectors for linearly dependent or independent if dependent find the relation

$X_1 = (2, 3, 4, -2)^T$, $X_2 = (-1, -2, -2, 1)^T$, $X_3 = (1, 1, 2, -1)^T$

b) Find the Eigen values of the following matrix and the Eigen vector corresponding to the smallest Eigen value

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

c) Find the characteristic equation of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ and show that the matrix A satisfies its characteristic equation.

d) Determine the Eigen values of the matrix $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ and hence determine the Eigen values of A^{-1} and A^4.

Q. 3. Attempt **ANY FOUR** (20)

a) Find all the values of $\left(1+i\sqrt{3}\right)^{3/4}$ and show that their product is 8.

b) Prove that $\frac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$.

c) If $5\sinh x - \cosh x = 5$ find $\tanh x$.

d) If $\sin(\theta + i\phi) = r(\cos\alpha + i\sin\alpha)$ then prove that $r^2 = \frac{1}{2}(\cosh 2\phi - \cos 2\theta)$.

e) If $\tan(x+iy) = i$ where x, y are real prove that x is indeterminate and y is infinite.

SECTION – II

Q. 4. Attempt **ANY THREE** (15)

a) Expand $e^{x\sin x}$ in powers of x upto x^4.

b) Prove that $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\left(n\pi + x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right)$

c) Express $3x^2 + 2x + 5$ in terms of $(x-2)$ by using Taylor's theorem.

d) Evaluate $\lim_{x\to 0}\left(\frac{x}{x-1} - \frac{1}{\log x}\right)$.

Q. 5. Attempt **ANY FOUR** (20)

a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

b) If $u = \operatorname{cosec}^{-1}\sqrt{\left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)}$ then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right).$$

c) If $f(x,y) = \left(50 - x^2 - y^2\right)^{1/2}$, find the approximate value of f(3, 4) – f(2.9, 4.1) by theory of approximation.

d) For the transformations $x = a(u + v)$, $y = b(u - v)$ and
$u = r^2 \cos 2\theta$, $v = r^2 \sin 2\theta$ find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

e) Divide 120 into three parts so that the sum of their product taken two at a time shall be maximum.

Q. 6. Attempt ANY THREE (15)

a) Solve by Gauss–Elimination method

$$3x + 4y + 5z = 18$$
$$2x - y + 8z = 13$$
$$5x - 2y + 7z = 20$$

b) Find the solution of the following system of equations using Jacobi's iterative method (Five iteration).

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

c) Solve using Gauss-Siedel method , the following system of equations

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

correct to 3 places of decimals.

d) Determine largest eigen value by iteration method of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$
